A Re-examination of the Laser-Supported Combustion Wave

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The basic equations and numerical solution procedures for a laser-supported "combustion" wave were re-examined with the objective of obtaining an unambiguous solution for the entire wave, including the region far downstream of the temperature peak. Correct boundary conditions far downstream of the wave were determined and a technique for numerical solution of the entire boundary value problem was developed. Solutions obtained for laser-supported combustion waves in hydrogen were compared with results of the shooting technique of Kemp and Root. The latter technique yields essentially the same relationship between incident laser intensity and mass flux through the wave as our complete solution. However, the Kemp-Root technique provides a solution for only part of the wave and is incapable, in general, of correctly predicting the fraction of incident laser intensity absorbed in the wave. The complete wave solution is also applicable to the propagation of laser-supported waves away from a fixed end wall at large times after the wave is initiated at the wall.

Introduction

LASER-sustained plasmas have been known since the advent of high-power lasers and were proposed as a means to heat a rocket propellant by Minovitch[1] and Kantrowitz.[2] Recent studies of laser-powered propulsion have been directed toward high-specific-impulse space propulsion systems for orbital transfer missions.[3,4] These studies have focused on the use of hydrogen as the primary propellant gas; and the analyses of rocket performance relied heavily on the concept of the laser-supported combustion (LSC) wave.[5,6]

Raizer[7] first drew the analogy between laser-sustained plasmas and combustion waves in his analysis of the subsonic propagation of light sparks in air produced by a weakly focused neodymium laser. He predicted that a continuous "optical plasmotron" could be produced using a continuous-wave (cw) carbon dioxide laser. The Raizer analysis was expanded upon by Jackson and Neilsen[8] and Havey,[9] and a two-dimensional extension was made by Batteh and Keefer.[10] The Raizer model was later applied to hydrogen by Kemp and Root,[5] who used it as a basis for the analysis of laser-powered rocket thrusters.[6] The two-dimensional model was extended to hydrogen plasmas by Keefer et al.[11]

One goal of all these analyses has been the prediction of the relationship between the intensity of the laser and the propagation velocity of the plasma. In all cases, the predicted velocities for the air plasmas are substantially less than those observed by Klosterman and Byron.[12] At present, there are no reliable measurements for hydrogen. Various mechanisms have been proposed to explain the discrepancy: radiation transport,[8] nonequilibrium radiation mechanisms,[9] and two-dimensional effects[10]; but incorporation of these mechanisms into models of the LSC wave has not as yet resulted in realistic predictions of the observed wave speeds. The relationship between laser intensity and propagation velocity is very important in the operation of a cw-laser-powered thruster, since the plasma must remain stationary in the absorption chamber.

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In the numerical approach used by Kemp and Root, LSC wave propagation is treated as an iterative initial value problem and the computations are made with a shooting technique. For a given value of incident laser intensity and mass flux through the wave, the calculated temperature distribution reaches a maximum and then starts to decay. At some distance downstream of the temperature peak, the computed temperature distribution diverges rapidly from the true solution. The temperature distribution either passes through a minimum before turning up or drops rapidly toward negative values. This branching behavior, which has the character of a saddle-point singularity, is assumed to have physical significance. The mass flux through the wave for a given incident laser intensity is taken to be that value determined, by iteration, to separate the two types of divergent solutions. With this approach, one cannot define the mass flux precisely enough to cause the solution to approach a finite asymptotic temperature far downstream of the wave. In fact, the branching behavior often occurs at temperatures high enough so that considerable laser absorption can occur downstream of the position at which the numerical solution diverges. Thus, in general, one cannot calculate the fraction of incident laser intensity absorbed in the LSC wave with the Kemp-Root approach.

The problems with the Kemp-Root approach prompted us to re-examine the Raizer model and to devise a numerical approach for the entire LSC wave in hydrogen that incorporates the proper boundary conditions far downstream of the wave.

The One-Dimensional LSC Wave Model

In the usual formulation of the problem, the LSC wave (represented as a sharp increase in temperature) propagates into the sustaining laser beam at constant pressure and speed. Radial components of the fluid velocity are neglected and the problem is assumed to be one-dimensional, although Raizer[7] includes an approximate radial conduction term and Kemp and Root[5] include the effect of radial transport of energy by radiation. The addition of these rather arbitrary radial energy transfer terms is a crude approach to modeling certain aspects of the actual two-dimensional structure of the LSC wave.

For a truly one-dimensional LSC wave, the continuity equation requires a constant-mass flux throughout the wave,

$$\rho u = \rho_0 u_0 = \text{const} \tag{1}$$

where ρ is the mass density, u is the fluid velocity, and the zero subscript refers to conditions far upstream of the LSC wave. The momentum equation is

$$u\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{2}$$

where x is the distance along the laser beam and p the pressure. Equation (1) states that if the density changes, then the velocity changes and Eq. (2) states that the pressure must also change. However, most investigators have ignored the pressure variation in the computation of the local density ($p/p_0 \simeq 1$) and the momentum equation is not included in the system of equations. The kinetic energy of the subsonic flow is usually neglected, but Kemp and Root[5] retain it, even though it is inconsistent with the assumption of constant pressure. That is, if the Mach number is low enough so that the variation in the static pressure is negligible, then the kinetic energy will also be negligible. Under the assumption of negligible kinetic energy, the energy equation can be written

$$\rho_0 u_0 C_p \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda \frac{\mathrm{d}T}{\mathrm{d}x}\right) + kI - \Phi \tag{3}$$

where C_p is the specific heat, T the temperature, λ the thermal conductivity, k the absorption coefficient for the laser radiation of intensity I, and Φ the net thermal radiation lost from the plasma. The intensity of the laser radiation is given by Beer's law,

$$\frac{\mathrm{d}I}{\mathrm{d}x} = -kI \tag{4}$$

A physically meaningful solution to Eqs. (3) and (4) must approach a finite temperature T_0 far upstream of the LSC wave and approach a finite temperature T_∞ far downstream of the wave (Fig. 1). Some insight into the nature of the solution can be gained by expressing the equations as an autonomous system of three first-order equations,[13]

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -\frac{q}{\lambda} \tag{5}$$

$$\frac{\mathrm{d}q}{\mathrm{d}x} = \frac{\rho_0 u_0 C_p}{\lambda} q + kI - \Phi \tag{6}$$

$$\frac{\mathrm{d}I}{\mathrm{d}x} = -kI \tag{7}$$

where q is the thermal flux. The critical points for this system are found when the three derivative terms vanish simultaneously,

$$q/\lambda = 0 \tag{8}$$

$$\rho_0 u_0 (C_p/\lambda) q + kI - \Phi = 0 \tag{9}$$

$$kI = 0 \tag{10}$$

C_p, λ, k, and Φ are temperature-dependent quantities and Eqs. (8-10) describe the mutual intersection of three surfaces in the three-dimensional phase space of the dependent variables T, q, and I. Since λ is finite, Eq. (8) yields one of these surfaces.

$$q = 0 \tag{11}$$

Equation (10) can be satisfied if either k or I is zero. In either case, substitution of Eqs. (10) and (11) into Eq. (9) gives

$$\Phi = 0 \tag{12}$$

Jackson and Nielsen[8] and Havey[9] performed detailed radiative transfer calculations to determine $\Phi(x,T)$, but most authors have assumed that the net radiative loss term is a function of temperature only. In the latter case, Eq. (12) implies that a second surface is the constant temperature,

$$T = T_\phi \tag{13}$$

at which the net thermal radiation loss is zero.

Two cases must be considered for Eq. (10). If the intensity is zero, this implies that all the laser energy has been absorbed and the critical point is

$$(T,q,I) = (T_\phi, 0, 0) \tag{14}$$

If the absorption coefficient is zero, this implies a threshold temperature T_k below which the plasma does not absorb. The critical point occurs at

$$(T,q,I) = (T_\phi, 0, I_\infty); \quad T_\phi < T_k \tag{15}$$

In either case, since a stable solution must pass through a critical point[13] as $x \to \infty$, Eqs. (14) and (15) show that the proper downstream boundary condition must be

$$T_\infty = T_\phi \tag{16}$$

If the net radiation Φ is considered to be a function only of temperature, then the correct model for the one-dimensional LSC wave consists of Eqs. (3) and (4) together with the boundary conditions,

$$T(-\infty) = T_0 \tag{17}$$

$$T(\infty) = T_\phi \tag{18}$$

$$I(-\infty) = I_0 \tag{19}$$

Numerical Solutions

A numerical method was used to obtain solutions to the one-dimensional LSC wave equations, together with the boundary conditions given by Eqs. (17-19). The solution domain was divided into two regions: the first extended from $x = -\infty$ to 0, taken to be the point at which the temperature in the downstream portion of the wave had dropped to T_k, the temperature threshold for absorption of laser radiation; the second extended from $x = 0$ to ∞. Different numerical methods were employed in each of these regions.

The first step was to obtain a solution for the region $x > 0$ where no laser absorption occurs. In this region, the problem is the boundary value problem

$$\rho_0 u_0 C_p \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\lambda \frac{\mathrm{d}T}{\mathrm{d}x}\right) - \Phi \tag{20}$$

$$T(0) = T_k \tag{21}$$

$$T(\infty) = T_\phi \tag{22}$$

which describes the relaxation of the downstream portion of the LSC wave as a result of convection, conduction, and radiation. The semi-infinite region was transformed to a finite region using the relation

$$\xi = 1/(1+x) \tag{23}$$

and the relaxation method of Patankar[14] was used to obtain a solution for a specified value of mass flux $\rho_0 u_0$.

Once this solution was obtained, the solution in the region $x<0$ was obtained using a fourth-order Runge-Kutta shooting method. The solution began at $x=0$ using the temperature T_k and the derivative of temperature $dT/dx|_{x=0}$ obtained from the computed solution in the region $x>0$. A guess was made for the intensity remaining in the laser beam after passing through the plasma $I(0)$. The solution proceeded toward $x=-\infty$ until the upstream temperature stabilized to a constant value or went negative. This procedure was repeated with a new value of $I(0)$ until the constant upstream value of temperature agreed with the desired upstream temperature boundary condition T_0. In this way, a complete numerical solution was obtained that satisfied both upstream and downstream boundary conditions. Once the solution was obtained, the incident laser intensity I_0 appropriate to the chosen mass flux was determined.

Results

Numerical calculations were performed for five separate cases of 10.6 μm carbon dioxide laser absorption in pure hydrogen at a pressure of 1 atm. The first four cases were obtained by neglecting the reabsorption of thermal radiation. The fifth case was computed using the conduction approximation for radiative transport proposed by Kemp and Root.[5]

All of the thermodynamic and transport properties for 1 atm hydrogen were fitted by cubic splines for use in the numerical computation. The specific heat and initial density values were obtained from Patch.[15] For the first four cases, the thermal conduction was taken from Grier[16] and, for the fifth, the thermal conduction was increased by addition of the radiative contribution given in the appendix of the paper by Kemp and Root.[5] The laser absorption coefficient for 10.6 μm radiation was computed from the inverse bremsstrahlung absorption of an equilibrium hydrogen plasma, using expressions given by Kemp and Lewis.[6] The equilibrium composition was calculated from the Saha equation using an empirical expression for the lowering of the ionization potential to provide agreement with the plasma composition given by Patch.[15] The net radiation term was also calculated from this plasma composition. The plasma was assumed to be optically thin for wavelengths greater than the Lyman alpha line (125 nm), with the exception of the H_α and H_β lines, which were assumed to be optically thick.

Both the laser absorption coefficient and the optically thin radiation loss are extremely strong functions of temperature in the region near the "threshold" for laser absorption and the choice of values for T_k and T_ϕ is, necessarily, somewhat arbitrary. The 8500 K temperature was chosen for the laser absorption threshold since at this temperature the laser absorption coefficient is approximately 10^{-3} times its maximum value. The 6000 K value for T_ϕ was chosen such that the optically thin radiation is approximately 10^{-3} times its value at T_k. In addition to the values of T_k and T_ϕ, it was also necessary to choose an input value for upstream flow velocity u_0. The results of these calculations are summarized in Table 1.

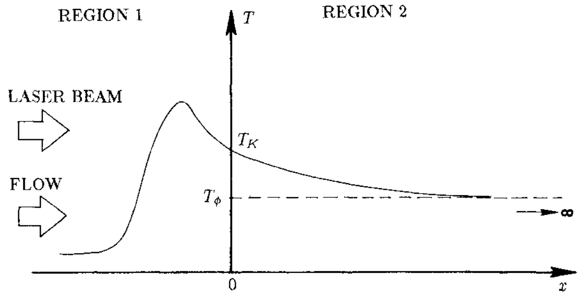

Fig. 1 Wave-fixed coordinate system with the origin fixed at the downstream position at which the wave temperature is T_k (separate numerical methods are used to obtain solutions in regions 1 and 2).

The temperature profiles for $u_0=5$ and 10 cm/s are shown in Figs. 2 and 3, respectively. Note that both the upstream and downstream temperatures are well behaved. The increase in velocity leads to a much steeper upstream temperature gradient and higher peak temperature. This is required for the thermal conduction to carry sufficient energy upstream to heat the incoming flow to a temperature at which the laser absorption is significant.

The $u_0=20$ cm/s case is shown in Fig. 4. The subroutines for the thermodynamic and transport properties were limited to temperatures below 25,000 K. This solution indicates that to keep the peak temperature less than 25,000 K for a flow velocity of 20 cm/s would require preheating of the incoming flow to a temperature of 5233 K.

One of the questions about the Kemp-Root numerical approach is whether the branching behavior in the temperature distributions can be used to calculate properly the incident laser intensity I_0 for a given mass flux. The Kemp-Root shooting technique was applied to the $u_0=5$ cm/s case. Curves 1-4 in Fig. 5 were obtained with the shooting technique and curve 5 is a portion of the complete solution already shown in Fig. 2. One can see that curves 3 and 4 seem to bracket the complete solution. However, there is a small discrepancy in the computed I_0 values. Curves 3 and 4 indicate that I_0 is between 9312.5 and 9325 W/cm³, while I_0 for curve 5 is 9389 W/cm³. This discrepancy is undoubtedly related to different numerical inaccuracies in the two computational methods and is probably not significant. It seems that the Kemp-Root shooting technique does yield the correct relationship between I_0 and mass flux through the LSC wave. Of course, only the complete solution will, in general, yield the correct fraction of the I_0 absorbed in the entire wave.

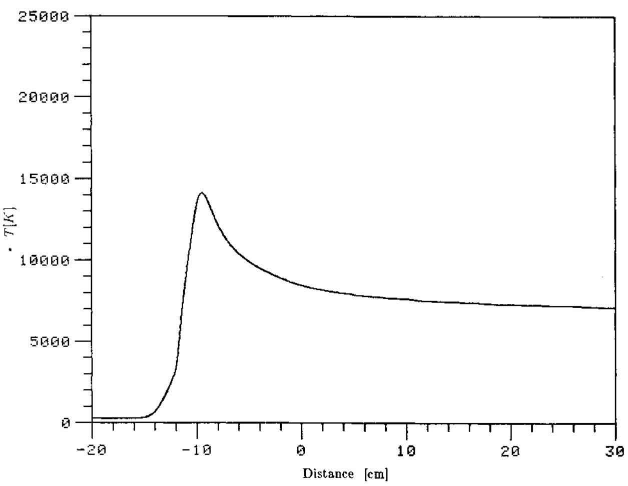

Fig. 2 Numerical solution for a propagation velocity of 5 cm/s in hydrogen at a pressure of 1 atm and temperature of 292 K. The incident intensity of the 10.6 μm carbon dioxide laser is 9389 W/cm² and 49.14% of the beam energy is absorbed.

Table 1 Summary of calculated results

Velocity u_0, cm/s	Intensity I_0, kW/cm²	Energy absorbed, %	T_0, K	Power absorbed, kW/cm²
2	7.84	26.8	289	2.1
5	9.39	49.1	300	4.6
10	13.34	74.3	314	9.9
20	18.21	87.6	5233	15.9
380	98.2	100.0	299	98.2

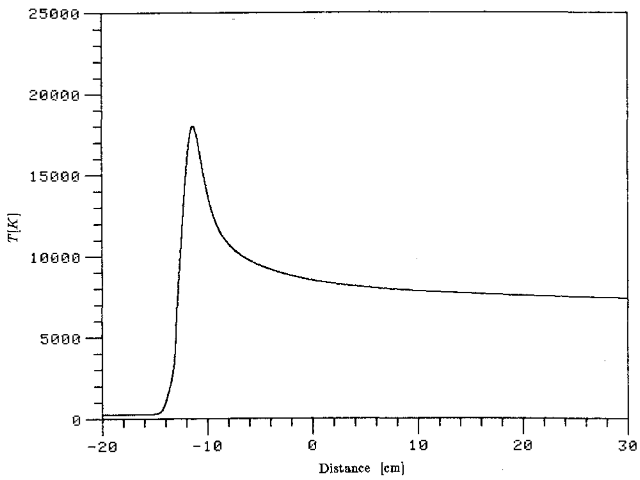

Fig. 3 Numerical solution for a propagation velocity of 10 cm/s in hydrogen at a pressure of 1 atm and temperature of 310 K. The incident intensity of the 10.6 μm carbon dioxide laser is 13.34 kW/cm^2 and 74.3% of the beam energy is absorbed.

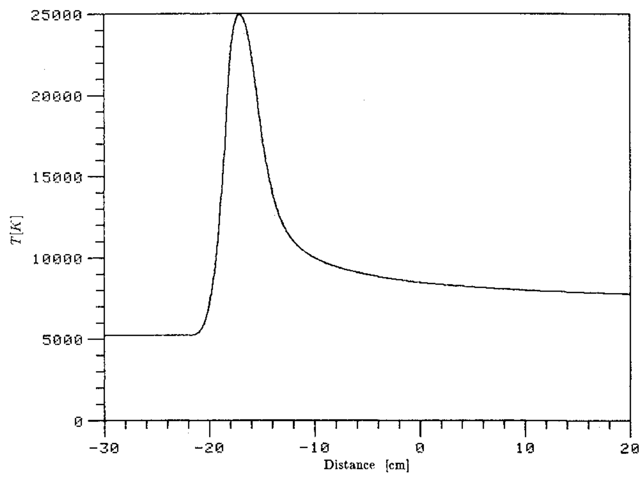

Fig. 4 Numerical solution for a propagation velocity of 20 cm/s in hydrogen at a pressure of 1 atm and a temperature of 5233 K. The incident intensity of the carbon dioxide laser is 18.2 kW/cm^2 and 87.6% of the beam energy is absorbed.

The cases discussed above were computed by neglecting radiative transfer. Jackson and Nielsen[8] performed detailed numerical radiative transfer calculations for the LSC wave in air utilizing 19 separate frequency intervals. On the basis of this calculation, they conclude that the conduction approximation to the radiative transport underestimates the upstream absorption peak by a factor of at least 40. However, Kemp and Root[5] used the conduction approximation for the numerical calculation of the LSC wave in hydrogen. The 380 cm/s case was chosen for comparison with their result. The effect of the conduction approximation is to increase the effective thermal conductivity λ. The effective value of thermal conductivity increases by more than four orders of magnitude between 10,000 and 20,000 K (Fig. 6). The large value of the thermal conductivity provides the necessary energy transport forward to heat the incoming gas without excessively high peak temperatures, as can be seen for the 380 cm/s case in Fig. 7. Comparison of this calculation with that of Kemp and Root[5] shows good agreement. The peak temperatures agree within 300 K and the incident intensity predicted by our calculation agrees to within 2% of the value

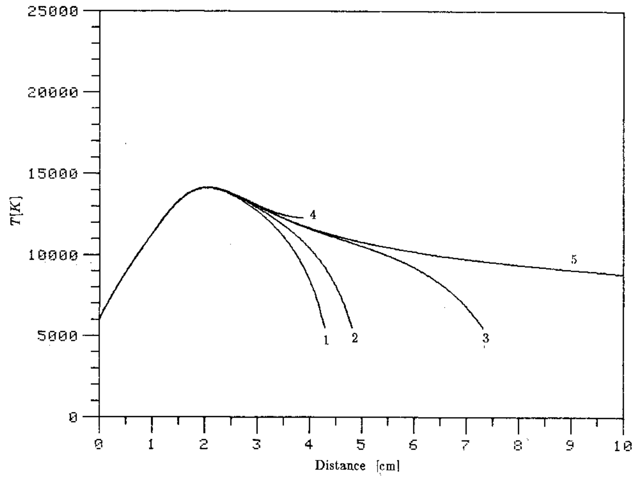

Fig. 5 Comparison of the numerical solutions obtained using the shooting method of Kemp and Root[1-4] and the complete solution of this paper.[5] This case is the same as that shown in Fig. 2.

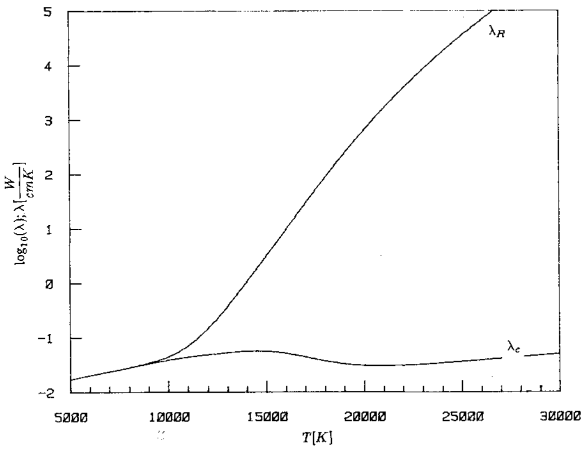

Fig. 6 Radiative λ_R and ordinary λ_c contributions to the total thermal conductivity for a 1 atm hydrogen plasma.

of 10^5 W/cm^2 used in the earlier calculation. The small difference is probably caused by the use of somewhat different values for the radiation loss.

Summary and Discussion

Examination of the autonomous system of differential equations that describe the one-dimensional LSC wave permits a priori determination of the boundary conditions far downstream of the wave and leads to an unambiguous computation procedure for the entire wave. Numerical solutions were obtained for CO_2-laser-sustained combustion waves in hydrogen at 1 atm. The results obtained indicate that the Kemp-Root shooting technique yields essentially the correct relationship between the incident laser intensity and the mass flux through the wave. In addition, our calculated temperature distributions in the forward portion of the wave agree reasonably well with those of Kemp and Root. Of course, the temperature distributions obtained with the complete solution do not diverge downstream of the temperature peak and the laser energy absorbed in the entire wave is predicted.

The computations were made in a wave-fixed coordinate system and yield the flow velocity u_0 far upstream of the wave for a given incident laser intensity I_0. By considering the boundary conditions far downstream of the wave in detail, we have obtained a solution that also applies to the

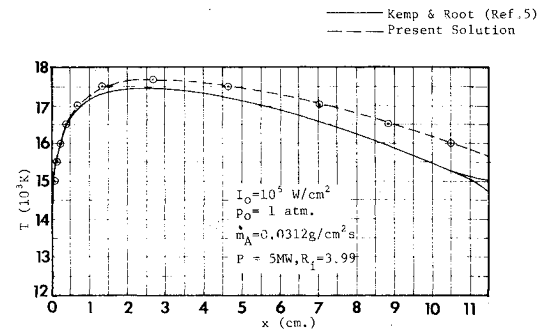

Fig. 7 Comparison between the published thermal profile given by Kemp and Root and the complete numerical solution of this paper.

asymptotic velocity of a one-dimensional subsonic LSC wave initiated at a fixed end wall in an initially quiescent gas. This is a large-time one-dimensional approximation of the type of experiment conducted by Klosterman and Byron[12] in air. Let us assume that the end wall is adiabatic as far as thermal conduction is concerned, but is transparent to radiation. Therefore, q at the wall is zero. Our analysis in wave-fixed coordinates indicates that $q(\infty)=0$ and $T(\infty)=T_\phi$. So, at relatively large times after initiation of the LSC wave at the end wall, when the wave is far from the wall, our solution satisfies the thermal boundary conditions at the wall. In wave-fixed coordinates, the flow velocity far downstream of the wave is u_∞. In laboratory coordinates, however, the velocity at the end wall is zero, so the entire wave-fixed reference frame must be translated away from the end wall at a fixed velocity of u_∞.

Since the mass flux through a steady one-dimensional LSC wave is constant, $u_\infty=\rho_0 u_0/\rho_\infty$, where ρ_∞ is the density of the gas at T_ϕ. For diatomic gases, ρ_0/ρ_∞ is typically 40, so the LSC wave at large times after initiation will propagate away from the end wall at about 40 times the speed of the cold gas relative to the wave. Of course, this means that the volumetric heating of the gas between the wave and the end wall causes the cold gas ahead of the wave to be given a velocity away from the wall equal to $(u_\infty - u_0)$.

In this discussion of the propagation of a one-dimensional LSC wave away from a fixed end wall, we have assumed the wall to be adiabatic as to thermal conduction and transparent to radiation. In fact, our prediction of the asymptotic LSC wave speed should be valid for any reasonable thermal boundary condition at the wall. At large times after initiation, the speed of propagation of a thermal disturbance caused by a nonadiabatic boundary condition at the end wall can be expected to be much smaller than the asymptotic speed of the LSC wave u_∞. The LSC wave simply outruns the effect of the thermal boundary condition at the end wall.

Our goal in this study was to clarify certain aspects of the one-dimensional LSC wave solution. Of course, we recognize that the condition of one-dimensionality will probably never be achieved, even approximately, in practice. Laser beam diameters are nearly always small in comparison to a characteristic thickness of the one-dimensional LSC wave. Consequently, radial transport of energy by conduction, convection, and/or radiation is undoubtedly significant in the propagation of actual LSC waves. Other factors leading to two-dimensionality in LSC waves are the convergence of focused laser beams and the nonuniformity of intensity across the beam. Neither an adequate prediction of the LSC wave propagation speeds measured by various investigators nor the rational preliminary design of LSC wave propulsion systems can be achieved until adequate two-dimensional models are developed.

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